Fluctuation-driven first-order behavior near the $T=0$ two-dimensional stripe to Fermi liquid transition

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The possibility is investigated that competition between fluctuations at different symmetry-related ordering wave vectors may affect the quantum phase transition between a fermi liquid and a longitudinal spin density wave state, in particular, giving rise to an intermediate "nematic" state with broken rotational symmetry but unbroken translational symmetry. At the marginal dimension the nematic transition is found to be preempted by a first-order transition but a weak symmetry breaking field restores a second-order magnetic transition with an intermediate regime in which correlations substantially enhance the broken rotational symmetry. Comparison to recent experiments is made.

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"Stripe" spin density wave order occurs in many high-T_c cuprate materials.[1–3] A spin density wave is a longitudinal modulation of the spin density $\vec{S}(\mathbf{R})$ characterized by a wave vector $\mathbf{Q}$ giving the periodicity of the spin modulation. In a "stripe," $2\mathbf{Q}$ is not a reciprocal lattice vector so the magnitude of the spin, as well as its direction, varies from lattice site to lattice site. If the underlying lattice has sufficient symmetry, stripe ordering may occur at one of several inequivalent wave vectors $\mathbf{Q}_a$. In the hole-doped high-T_c cuprates the important physics is two dimensional and the lattice has (to a good approximation) square symmetry. For dopings greater than about $x=0.05$ and less than a material-dependent number ranging from 0.08 [in $YBa_2Cu_3O_{6+x}$ (Ref. 4)] (YBCO) to 0.24 [in $La_{1.6-x}Nd_{.4}Sr_xCuO_4$ (Refs. 5–7)] order is believed to occur at one of the two wave vectors $\mathbf{Q}_x=(\pi-\delta,\pi)$ or $\mathbf{Q}_y=(\pi,\pi-\delta)$ with δ doping dependent and generically nonzero.[1,2,4,7–9]

In a stripe state the expectation value of the spin density at position $\mathbf{R}$, $\vec{S}(\mathbf{R})$, may be written as

$$\langle\vec{S}(\mathbf{R})\rangle=\vec{A}_a\cos(\mathbf{Q}_a\cdot\mathbf{R}+\theta_a). \tag{1}$$

The state defined by Eq. (1) breaks spin rotation, lattice translation and lattice rotation symmetries. A phase characterized by a nonvanishing $\langle\vec{S}(\mathbf{R})\rangle$ is also characterized by nonvanishing

$$\langle\mathcal{T}(\mathbf{R})\rangle\equiv\langle\vec{S}(\mathbf{R})\cdot\vec{S}(\mathbf{R})\rangle-\langle\langle\vec{S}(\mathbf{R})\cdot\vec{S}(\mathbf{R})\rangle\rangle \sim\cos(2\mathbf{Q}_a\cdot\mathbf{R}+2\theta_a) \tag{2}$$

where the double-bracket indicates also an average over position. $\mathcal{T}$ is spin rotation invariant but breaks lattice translation and rotation symmetry if $2\mathbf{Q}$ is not a reciprocal lattice vector. (One may also consider bond order involving $\langle\vec{S}(\mathbf{R})\cdot\vec{S}(\mathbf{R}'\neq\mathbf{R})\rangle$ but this will not be important here). $\mathcal{T}$ couples linearly to lattice distortions and the electronic charge density, so is observable in scattering measurements[1,8,10] and is sometimes referred to as "charge order."

The "stripe" state is also characterized by nonvanishing

$$\langle\eta(\mathbf{R})\rangle=\langle\langle\vec{S}_{Q_x}(R)\cdot\vec{S}_{Q_x}(R)-\vec{S}_{Q_y}(R)\cdot\vec{S}_{Q_y}(R)\rangle\rangle \tag{3}$$

where S_{Q_a} indicates spin fluctuations with wave vectors near Q_a. $\langle\eta\rangle$ is invariant under spin rotations and lattice translations but breaks the discrete lattice rotation symmetry and may be referred to as a nematic order parameter.[8,10]

The three broken symmetries may be restored at separate transitions.[8,10] (Very similar phenomena are well understood in the classical physics context of smetic and nematic liquid crystals.[11]) If spin order is destroyed by fluctuations in the direction of $\vec{A}$ (as would happen in a two dimensional model with Heisenberg symmetry at any $T>0$), $\langle\mathcal{T}\rangle$ and $\langle\eta\rangle$ may be expected to remain nonzero. If long-ranged order in T is destroyed by fluctuations in θ, $\langle\eta\rangle$ may remain nonvanishing. In physical terms, the state with $\langle\vec{S}\rangle=\langle\mathcal{T}\rangle=0$ but $\langle\eta\rangle\neq0$ has the property that fluctuations around one of the $\mathbf{Q}_a$ are larger than fluctuations around the other possible $\mathbf{Q}_{b\neq a}$.

Experimental evidence suggests that this sequence of transitions indeed occurs in some high T_c compounds. Many measurements[1,8] indicate that in underdoped cuprates the ground state (if superconductivity is suppressed) is characterized by magnetic scattering at the wave vectors $\mathbf{Q}_{x,y}$ and $2\mathbf{Q}_{x,y}$ but not at $\mathbf{Q_x}\pm\mathbf{Q_y}$, implying that the scattering signal arises from a superposition of domains with order at either $\mathbf{Q}_x$ or $\mathbf{Q}_y$. As temperature is raised above an ordered state the Bragg scattering at $\mathbf{Q}$ vanishes first, leaving an intermediate state with Bragg scattering only at $\mathbf{2Q}_x$ and $\mathbf{2Q}_y$.[1] Recent neutron scattering measurements[4] on a monodomain sample of $YBa_2Cu_3O_{6.45}$ indicate a wide temperature regime where there is order neither at $\mathbf{Q}_{x,y}$ nor at $2\mathbf{Q}_{x,y}$ but where the fluctuations associated with ordering wave vector $\mathbf{Q}_x$ have much longer spatial range and stronger temperature dependence than the fluctuations associated with potential ordering wave vector $\mathbf{Q}_y$. Transport measurements have detected rotational symmetry breaking[2] and, recently, an enhancement of the Nernst effect in this temperature regime has been reported,[9,12] also consistent[13] with an intermediate nematic phase. Similar transport behavior in the $(La/Nd)_{2-x}Sr_xCuO_4$ family of materials has also been interpreted in terms of a nematic phase or regime.[6] It is however important to note that the crystal structure of both of these materials is such that a CuO_2 plane is orthorhombically distorted so it may be more appropriate to describe the observed "nematic" regime as being characterized by a strong and strongly doping and temperature-dependent enhancement of a preexisting anisotropy.

Closely related issues have been discussed in the context of the pnictide materials[14,15] where $2\mathbf{Q}$ is a reciprocal lattice vector[16] so only the spin and nematic orders are relevant. Also in pnictides a strong coupling to lattice distortions believed to be important.

These and related experiments have focused theoretical attention on "nematicity." A number of works consider nematic phases which are taken to be conceptually independent of any density wave ordering.[13,17–24] This paper considers the density wave instability as primary with the nematic phase arising from it. The physical idea is straightforward: in a stripe situation, density wave ordering at one possible ordering wave vector $\mathbf{Q}_a$ must act to suppress density wave ordering at the other possible wave vectors $\mathbf{Q}_{b\neq a}$. Thus, as a putative "stripe" quantum critical point is approached competition between fluctuations at different wave vectors may drive a "nematic" transition at which the system chooses one wave vector at which the fluctuations will become critical, while fluctuations at the other wave vectors remain massive. Alternative possibilities are that the competition is important only for selecting the relevant state inside the density wave ordered phase, or that competition between fluctuations may drive the transition first order. In renormalization group language the question is whether there is a relevant operator at the stripe critical point and, if so, does it imply a flow to a new "nematic" critical point or a runaway flow indicating a first-order transition.

This paper approaches the physics in terms of a $T=0$ instability of a disordered fermi-liquid phase using the standard "Hertz" model of a density wave transition in a two-dimensional fermi liquid.[25,26] The main finding is the phase diagram depicted in Fig. 1: for lattices with square symmetry the quantum "nematic" transition is typically preempted by a strongly first-order transition directly to a density wave state; however, a weak explicit symmetry breaking restores a continuous transition.

Incommensurate density wave transitions are the subject of an extensive literature,[27,28] but the issue of interest here has been less studied. Physics similar to that of interest here has been explored in the context of classical spin models for cuprates[29] and pnictides.[14] Also, although their main focus was on transitions between nematic and density wave ordered states, Sun and co-workers observed[22] that the basic fermi liquid to density wave transition would likely be first order. DePrato *et al.*[30] used renormalization group techniques to classify the quantum critical fixed points of a model of "stripe" quantum criticality involving undamped spin excitations in two spatial dimensions, in particular identifying and analyzing regimes of stable second-order transitions. At these transitions there would be no intermediate nematic phase separating the density wave and disordered state.

Pelissetto *et al.*[31] studied a spin density wave transition occurring inside a d-wave superconducting state. The most relevant perturbation involved coupling of nodal fermions to a nematic order parameter derived from the spin fluctuations

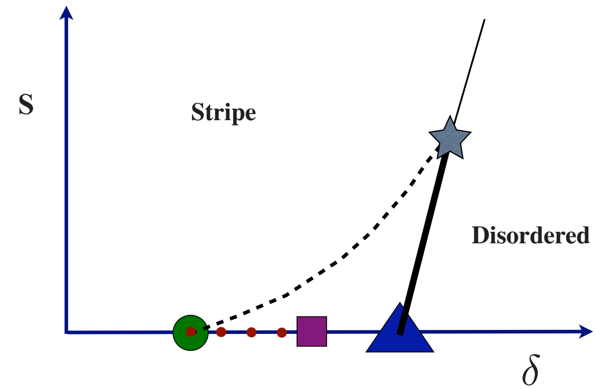

FIG. 1. (Color online) Zero temperature phase diagram of two dimensional stripe ordering model in plane of doping δ and tetragonal symmetry breaking parameter S. Solid lines: phase transition between paramagnetic metal ("disordered") and density wave ("stripe") state. Star (light blue online): tricritical point separating first-order (heavy line) and second-order (light line) phase transitions. Dashed line (black online); putative second-order density wave transition preempted by first-order transition. Small filled circles (red online): paramagnetic nematic phase, defined only at $S=0$ and separated from density wave and disordered phases by second-order phase transition points marked, respectively, by large circle (green online) and square (purple online). Nematic phase and associated transitions are also preempted by first-order transition

in a manner very similar to what is considered here; however again the density wave transition was not preempted by a nematic one. Qi and Xu[15] studied a spin-fermion model, finding a runaway flow also indicating first-order transitions, but only at an exponentially long length scale.[24]

A theory of the disordered spin density wave may be obtained from Eq. (1) by regarding $\vec{A}$ and θ as slowly fluctuating quantities. We specialize to the two dimensional square (or rectangular) lattice. Because we are interested in the transition from a fermi liquid where all spin amplitudes are small we combine A_a and θ_a into a new complex field $\vec{\psi}_a(r)=\vec{A}_a(r)e^{i\theta_a(r)}$ which we assume is described by the action $S=S_{dyn}+S_{static}$ with

$$S_{static}=\int d^2rd\tau\sum_{a=x,y}\left(\frac{1}{2}|\nabla\vec{\psi}_a|^2+\frac{1}{2}\delta_a|\vec{\psi}_a^2|\right)+\int d^2rd\tau\frac{u+v}{8}(|\vec{\psi}_x^*\cdot\vec{\psi}_x|+|\vec{\psi}_y^*\cdot\vec{\psi}_y|)^2+\frac{u-v}{8}(|\vec{\psi}_x^*\cdot\vec{\psi}_x|-|\vec{\psi}_y^*\cdot\vec{\psi}_y|)^2 \quad (4)$$

The δ_a are control parameters (for example, doping) which tune the system through the magnetic quantum critical point and we have allowed for the possibility that deviations from tetragonal symmetry favor ordering in one direction rather than another. Because the experimental evidence in high-T_c materials indicates that spin fluctuations are strongly peaked near discrete momentum values we do not need to consider the possibility of a continuous rotation of the wave vector and any potential nematic phase would have a strong Ising anisotropy.

Of the six possible quartic nonlinearities (see Ref. 30 for a complete list) Eq. (4) includes only the two which are important for the present purpose, neglecting terms which favor spiral and other nonstripe states or renormalize the basic stiffness against large amplitudes of the fields. The crucial term is the third one, which quantifies the extent to which orders in the x and y directions compete with each other. The relevant case is $v>u$, so that fluctuations compete. On the mean field level stability of this theory requires that $u>0$ and $v>-u$.

We assume standard overdamped dynamics. For ease of writing we present S_{dyn} in frequency space,

$$S_{dyn} = \frac{1}{2}\sum_{a=x,y} T\sum_{n} \int d^2r \frac{|\Omega_n|}{\Gamma}|\vec{\psi}^a \cdot \vec{\psi}^a|. \tag{5}$$

The theory requires an ultraviolet cutoff. We measure energy in units of Γ and impose a hard cutoff, eliminating all processes for which $|\Omega|/\Gamma + k^2 > \Lambda$. Qi and Xu[15] studied essentially this model, but with an additional $(\vec{\psi}_1 \cdot \vec{\psi}_2)^2$ coupling.

As defined the upper critical dimension of the model is $d=2$ and the physics may be studied by a renormalization-group analysis. The required beta functions are given in Eq. (3.1) of Ref. 30. It is useful define new variables g and ϕ by $u=g\cos\phi$, $v=g\sin\phi$ which flow according to (here the dot denotes changes with renormalization group cutoff parameter)

$$\dot{g} = -(7\cos^3\phi + 11\cos\phi\sin^2\phi + 2\sin^3\phi)g^2, \tag{6}$$

$$\dot{\phi} = (3\sin^3\phi - 2\sin^2\phi\cos\phi - \sin\phi\cos^2\phi)g. \tag{7}$$

Because $\dot{g} \sim g^2$ while $\dot{\phi} \sim g$, ϕ flows much more rapidly than g and the content of the theory may be understood from a constant g. In Eq. (7) the angle $\phi/4$ (corresponding to $u=v$) is a separatrix. For $\phi < \pi/4$ the flow is toward $\phi=0$, but in the $\phi > \pi/4$ case relevant to stripe physics the flow is toward a fixed point value which is close to π. Noting that u turns negative at $\phi=\pi/2$ we see that the renormalization group analysis indicates that when the flow passes this point the two dimensional stripe fixed point becomes unstable toward a first-order transition. The basic conclusion is perhaps not surprising: the model of two coupled order parameter fields is a textbook example of a runaway flow leading to a first-order transition.[32,33] De Prato *et al.*[30] observed that their more general renormalization group equations had only unstable fixed points near the marginal dimension and Sun *et al.* noted that multicritical points of this type tend to be unapproachable due to the presence of runaway flows.

Qi and Xu[15] similarly noted the possibility of a runaway flow. The equations of Ref. 15 involve three couplings and are thus more complicated to solve. A numerical solution was presented which indicated a runaway flow, albeit beginning at an exponentially low scale, whereas what is found here is a first-order transition at a scale which is not, in general, exponentially small.

The first-order nature of the transition arises from competition between fluctuations associated with the two ordering wave vectors. An explicit symmetry breaking term (arising, e.g., from the chains in YBCO) would grow under renormalization and if it became large enough, would quench the fluctuations at one of the two wave vectors, thereby permitting a continuous behavior. To understand the energy scales involved in this scenario we consider a self-consistent one-loop analysis, which while less rigorous than a renormalization group treatment has a transparent physical interpretation and allows for straightforward estimations of energy scales.

To implement the self consistent one loop theory we write the model as a functional integral and decouple the nonlinearities $|\psi_x|^2+|\psi_y|^2$ and $|\psi_x|^2-|\psi_y|^2$ by Hubbard-Stratonovich fields $i\lambda$ and η respectively and then integrate over the ψ fields obtaining the action

$$S[\lambda,\eta] = \frac{\lambda^2}{2(v+u)} + \frac{\eta^2}{2(v-u)} + \frac{3}{2}\mathrm{Tr}\ln[\Pi_0 + \delta_x + i\lambda + \eta] + \frac{3}{2}\mathrm{Tr}\ln[\Pi_0 + \delta_y + i\lambda - \eta] \tag{8}$$

with $\Pi_0=|\Omega|+k^2$. Equation (8) is written for the paramagnetic phase; the factor of 3 is the spin degeneracy. Mean field theory corresponds to finding the λ^* and η^* which extremize S. The extremal values of λ are imaginary; we write $i\lambda=\bar{\delta}-r$ with $\bar{\delta}=(\delta_x+\delta_y)/2$ and introduce $\Delta=(\delta_x-\delta_y)/2$ which parametrizes any explicit breaking of tetragonal symmetry. The correlation length $\xi_{x,y}$ for spin fluctuations around the wave vectors $\mathbf{Q}_{x,y}$ is $\xi_{x,y}^{-2}=r\pm(\eta+\Delta)$. In the paramagnetic phase $\xi_a^{-2}>0$. If one of the fields, say ψ_y, orders, then the Heisenberg symmetry ensures that the two transverse components are gapless ($\xi_{\perp,y}^{-2}=0$) while the longitudinal component has a correlation length determined by the magnetization m, ($\xi_{\parallel,y}^{-2}=m^2$) so that the term $\frac{3}{2}Tr\ln[\Pi_0+\delta_y+i\lambda-\eta] \to Tr\ln[\Pi_0]+\frac{1}{2}Tr\ln[\Pi_0+m^2]$.

Fang *et al.*[14] presented a large-N analysis of a classical spin model which leads to a theory very similar to that defined by Eq. (8) if $\bar{\delta}$ is chosen to be deep in the ordered phase so quantum fluctuations are unimportant.

Defining δ_{crit} to be the value at which S is extremized at $\Delta=r_x=r_y=0$, redefining $\delta_{x,y}$ as the difference from δ_{crit}, introducing $\bar{u}(\bar{v})=3u(v)/4\pi^2$ and explicitly carrying out the minimization in the paramagnetic phase at $T=0$ we obtain

$$\delta_x = r_x\left(1+\bar{u}\ln\frac{1}{r_x}\right) + \bar{v}r_y\ln\frac{1}{r_y}, \tag{9}$$

$$\delta_y = r_y\left(1+\bar{u}\ln\frac{1}{r_y}\right) + \bar{v}r_x\ln\frac{1}{r_x}. \tag{10}$$

In the tetragonal symmetry case $\Delta=0$ and at $\bar{\delta}>0$ Eqs. (9) and (10) admit an isotropic solution $r_x=r_y>0$ corresponding to the conventional paramagnetic phase. However, for $v>u$ one finds that as δ is decreased below a critical value $\delta_{nem}>0$ the isotropic solution undergoes a bifurcation to a solution with $r_x\neq r_y$. (Fang *et al.*[14] found a very similar transition, in their case thermally driven.) This is a nematic phase: the nematic order parameter is $\eta\sim\langle|\psi_x|^2\rangle-\langle|\psi_y|^2\rangle$. Writing $r_x=r+\eta/2$, $r_y=r-\eta/2$, and linearizing in η we find

$$\delta_{nem} = \frac{2\bar{v} + \bar{v}^2 - \bar{u}^2}{\bar{v} - \bar{u}} e^{-1-1/(\bar{v}-\bar{u})} \tag{11}$$

However, within mean field theory this critical point is typically preempted by a first-order transition to a state with long-ranged stripe order. The first-order transition manifests itself as a failure of numerical routines to find a solution to Eqs. (9) and (10) as δ is decreased below a spinodal value greater than δ_{nem} and may also be seen more directly.

We have computed the energy of a magnetized state by generalizing Eq. (8) as described above. We find that at $\delta = \delta_{nem}$ an ordered state with $r = \pm\eta$ and $m > 0$ exists and has lower energy than the paramagnetic state, provided that $\bar{v}$ is not too large. Thus at some $\delta > \delta_{nem}$ the system will jump from the isotropic paramagnetic phase to a phase with long-ranged order. The largest δ at which an ordered phase may be sustained may be ascertained from the δ at which there is a solution to Eqs. (9) and (10) with $r_y = 0$ and $r_x > 0$. This $\delta = \delta_{1^{st}-order}$ is

$$\delta_{1^{st}-order} = \frac{\bar{v} e^{-1/(\bar{v}-\bar{u})}}{\bar{v} - \bar{u}} = \frac{e}{2 + \bar{v} - \frac{\bar{u}^2}{\bar{v}}} \delta_{nem}. \tag{12}$$

By comparison of energies we find that if $\delta_{1^{st}-order} > \delta_{nem}$ the nematic transition is preempted by a first-order transition. As the second equality of Eq. (12) shows, for $\bar{v} > v_c(u)$ with $v_c(u=0) = e - 2 \approx 0.718\ldots$, $\delta_{nem} > \delta_{order}$ so that a second-order transition can exist at large v. However, in the large $\bar{v}$ regime in which the transition (within the present theory) is second order, substitution into the defining equations shows that the renormalized mass r is of the order of the cutoff, indicating that the second-order transition occurs in a regime beyond the range of validity of the critical theory. We interpret this result as meaning that the nematicity arising in the large v case is an intrinsic phenomenon arising from short length scale physics and not directly related to the singular magnetic critical fluctuations.

We are now in a position to consider the effects of an explicit breaking of the C_4 lattice rotational symmetry, such as occurs in $YBa_2Cu_3O_{6+x}$ and $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$. The effect of an explicit breaking of the C_4 symmetry on the nematic order is analogous to the effect of a magnetic field on ferromagnetic order: it converts a (hypothetical) second-order transition into a smooth crossover. More importantly, breaking C_4 symmetry means a relative suppression of fluctuations near one of the two possible ordering wave vectors and relative enhancement of fluctuations near the other. Because the first-order transition was induced by competition of fluctuations, this will tend to convert the first-order transition to a continuous one The small value of $(\delta_{1^{st}-order} - \delta_{nem})/\delta_{nem}$ suggests that the symmetry breaking field needed to change the order of the transition will not be large. Figure 2 presents results obtained by solving Eqs. (9) and (10) for $\delta_x = \delta_y + \Delta$ for a representative choice of parameters $\bar{u} = 0.3$ and $\bar{v} = 0.6$ (implying $\delta_{1^{st}order} = 0.071$ and $\delta_{nem} = 0.0328$). and several levels of anisotropy. One sees that for these parameters anisotropy greater than about 0.01 $\sim (\delta_{1^{st}order} - \delta_{nem})/4$ restores a continuous behavior.

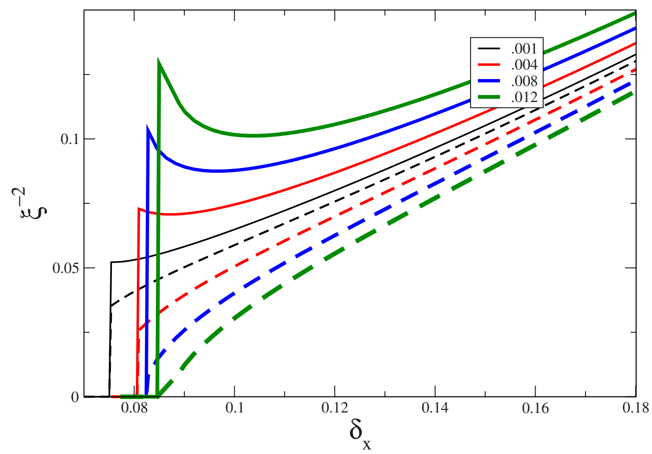

FIG. 2. (Color online) Dependence on control parameter δ of inverse correlation length parameters for x (solid line) and y (dashed line) wave vectors for different levels of explicit symmetry breaking $\delta_x = \delta_y + \Delta$ with anisotropy parameters Δ as indicated and interactions $\bar{u} = 0.3$ and $\bar{v} = 0.6$.

Extending the results to temperatures $T > 0$ is complicated by issues including the difficulty of treating the classical transitions below their upper critical dimension, the interplay between "nematic" and $2\mathbf{Q}$ ("charge") order and the possible first-order nature of the transitions. A detailed discussion will be given elsewhere but a few remarks are in order here.

One should distinguish two situations: either the model has an intrinsic tendency toward nematic behavior, not directly related to the stripe criticality (in the model defined by Eq. (4) this would occur if v is large, both absolutely and relative to u) or the first-order transition preempts the $T=0$ nematic phase. Models with "intrinsic" nematicity have been extensively discussed elsewhere[13,17–23] and will not be considered here. In the case where the $T=0$ nematic phase is preempted by a first-order transition there remains the possibility of nematic behavior at $T > 0$.

Indeed the model defined in Eq. (4) trivially exhibits nematic behavior if parameters are chosen so that the model is in the magnetically ordered state at temperature $T=0$ and if charge order is not important (for example because $2\mathbf{Q}$ is a reciprocal lattice vector). The ordered state must select one of the two wave vectors, say $\mathbf{Q}_x$. The Heisenberg symmetry and two dimensionality then implies that if temperature is slightly raised above the ordered state, fluctuations near $\mathbf{Q}_x$ have a correlation length of the "renormalized classical" form $\xi \sim \exp[\rho_s/T]$ while fluctuations near $\mathbf{Q}_y$ have a relatively short and weakly temperature dependent correlation length, so the resulting state is a "nematic." The question then is whether this "thermal spin nematic" behavior vanishes via a first or a second-order transition. If a weak interlayer coupling is added to the model one may ask if the nematic behavior survives at temperatures higher than the three dimensional ordering temperature (this issue was addressed in the classical model of Ref. 14). If charge order is also important, one may ask if the nematic behavior vanishes at the charge ordering temperature or at a higher temperature, and what are the orders of the transitions.

Clearly a transition which is first order at $T=0$ remains first order as the phase boundary is extended to $T>0$, at least within some distance of the zero temperature transition, so an extension of the present results to $T>0$ would suggest a first-order thermal transition, at least for dopings near where the $T=0$ ordered state vanishes, but it is not ruled out that the transition may become second order as doping is reduced deeper into the ordered phase.

The self-consistent one-loop approximation employed above leads to a reentrant behavior. As T is increased from zero the first-order transition line extends to the larger δ side of the critical point and the transition is from an isotropic paramagnetic phase at low T to a higher T "nematic renormalized classical" phase in which the spin fluctuations near one of the two ordering wave vectors have a very long and rapidly temperature-dependent correlation length while those near the other have a correlation length which is relatively short and weakly temperature dependent. Whether the "nematic renormalized classical" phase is also characterized by long-ranged $2\mathbf{Q}$ charge order depends on details. As the temperature is further increased the charge order (if any) disappears and there is a second transition (typically also first-order within this approximation) to a fully isotropic phase. The first-order transition persists as parameters are tuned to move deeper into the insulating phase. While the reentrant behavior seems unphysical, and is likely to be an artifact of the approximation, the qualitative result of a range of zero temperature parameters above which the thermal transition is first order is robust.

The classical-spin results of Ref. 14 provide an interesting perspective on this issue. As remarked above, this model is in essence the self-consistent one-loop approximation to the classical limit of the model defined in Eq. (8) for δ deep within the $T=0$ ordered phase and $2\mathbf{Q}$ a reciprocal lattice vector. The authors of Ref. 14 considered a regime in which the physics was controlled by the "renormalized classical" divergence of the correlation length and moreover chose parameters corresponding to an extremely weak tendency toward nematic ordering. Reference 14 defines a dimensionless quantity $\tilde{K}/\tilde{J}_2$, which is essentially $(v-u)/(v+u)$ in the notation of this paper. This was taken to have the very small value 7.5×10^{-3}. For this parameter value Ref. 14 reported a second-order "nematic" transition which occurred at very low temperatures deep in the "renormalized classical" regime and was not preempted by a first-order transition. Reference 14 did not report results for larger $\tilde{K}$, but solving their equations indicates that as $\tilde{K}$ is increased to a value $\gtrsim 0.075\tilde{J}_2$ the thermally driven transition again becomes first order. Now $4\pi\tilde{J}_2$ is essentially the spin stiffness of the magnetically ordered state which may be thought to grow with distance into the ordered phase so if one imagines that $\tilde{K}\sim(v-u)$ is fixed to a value which is not too large, the increase in the magnetic spin stiffness distance into the ordered phase may eventually drive the model into the second-order transition regime.

Of course, the reliability of the self-consistent one-loop approximation may be questioned, especially for thermal phenomena. The analysis of Ref. 26 indicates that this analysis is essentially equivalent to a renormalization group analysis for models above the upper critical dimension, but for models which are below the upper critical dimension this is of course not the case. The self-consistent one loop approximation has the exponents of the Gaussian model, and for this reason may overestimate the tendency toward nematic order. For example, one may define a "nematic susceptibility" via the correlation function $\langle(\psi_x^2-\psi_y^2)^2\rangle$. This correlator is closely related to the energy correlator that defines the specific heat exponent. The Gaussian model dramatically overestimates the divergence of the specific heat near two-dimensional and three-dimensional classical transitions and may well similarly overestimate the divergence of the "nematic susceptibility."

Despite these difficulties it is interesting to relate the picture presented here to data. The essential point is that in models in which "nematicity" is derived from competition between density wave ordering at several wave vectors, the nematic phase may be preempted by a first-order transition, but if the lattice rotational symmetry is explicitly broken a more continuous behavior may be restored. There is no direct evidence that stripe order terminates at a first-order quantum critical point in any cuprate, although we note that phase coexistence, a typical consequence of first-order phase boundaries, is common in cuprate materials. The strongest evidence in favor of a nematic regime comes from the YBCO and $(La/Nd)_{2-x}Sr_xCuO_4$ families of materials[4,9,12] where the lattice symmetry is explicitly broken. It is very tempting to argue that the symmetry breaking pushes the system into the continuous transition regime identified above. It may also be worthwhile to reexamine the data to determine if a local lattice distortion (favoring one or the other wave vector) is present or if a hysteresis has been overlooked.

In summary, this paper has posed the question of the existence of an intermediate nematic phase separating a stripe and a fermi liquid phase in terms of fluctuation corrections to a putative "stripe" quantum critical point. The physics that can lead to a nematic phase also leads naturally to a fluctuation-driven first-order transition, which near the quantum critical point was found to preempt the nematic phase. The first-order transition is not inevitable. If parameters were tuned so that the nematic transition occurs "far" from the putative stripe quantum critical point so that the "nematicity" is an intrinsic effect and not driven by critical density wave fluctuations, then the considerations of this paper are not relevant.

It was found that the first-order transition could be converted to second order by a quite small anisotropy. The analysis relied on approximations including the self-consistent one-loop theory (which is uncontrolled) and the Hertz quantum critical theory (which is subject to corrections whose nature remains incompletely understood[27]) but the first-order behavior discussed here follows from relatively general considerations and seems likely to be robust. In systems where nematic behavior was found a reexamination of experimental data for signatures of first-order transitions (for example, hysteresis and phase coexistence) may be worthwhile. Extension of the results presented here to $T>0$, to include coupling to the lattice, and to other situations, such as the metamagnetic transition in $Sr_3Ru_2O_7$, would be of interest.

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